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AN EVOLUTIONARY ALGORITHM TO ESTIMATE UNKNOWN HEAT FLUX IN A ONE-DIMENSIONAL INVERSE HEAT CONDUCTION PROBLEM

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Abstract – A one-dimensional inverse heat conduction problem (IHCP) of estimating an unknown heat flux, given experimental temperature measurements at a point in the domain, is solved using evolutionary algorithms (EA). The EA based methods used to solve the one-dimensional IHCP in the literature involve minimization of the Tikhonov functional. In this paper, a modified approach is utilized wherein the first term of the Tikhonov functional is assigned a weight depending on the time step. More weight is assigned initially, and this weight is reduced gradually so that the EA pays more attention to the initial heat flux values. This is required because it would be extremely difficult to estimate the later unknown heat flux values correctly without estimating the initial unknown heat flux values correctly. This enables the EA to move quickly to find the minimized values. This approach performs better in terms of the number of mating events to solution when compared with that without using any weights, in the case of the one-dimensional IHCP examined.

NOMENCLATURE

α	Thermal diffusivity
β	Regularization parameter
T	Temperature
t	Time
x	Spatial variable along x direction
q	Heat flux
Δt	Time step
Δx	Distance along grid points in x direction
λ	$\frac{\alpha \cdot (\Delta t)}{(\Delta x)^2}$
g	Spatial variation of initial temperature in the domain
f	Fitness function for minimization problem
f'	Fitness function for maximization problem
N	Number of grid points in the domain
$w(t)$	Weight function

Subscripts and Superscripts

i	Spatial grid index
j	Time index
meas	Values that have been measured
cand	Values from a candidate solution

1. INTRODUCTION

The inverse heat conduction problem (IHCP) has been widely studied in the literature [1-3]. Several analytical approaches have been used to solve the IHCP. Also, optimization techniques like the conjugate gradient method have been used. In addition to the analytical and gradient based optimization based techniques, artificial intelligence techniques like genetic algorithms and neural networks have also been used within an optimization framework as solution techniques for the IHCP. Mera *et al.* [9] have studied the use of genetic algorithms in the solution of ill-posed problems and have pointed out that additional constraints have to be incorporated to stabilize the solution and that an EA does not have a self-regularizing property. Raudensky *et al.* [5] used a genetic algorithm to solve the non-linear IHCP. In this paper, an evolutionary technique is discussed in which weights are assigned to the fitness function. It has been found that assigning weights has increased the average fitness of the population during the evolutionary process. A linear, one-dimensional heat conduction problem has been solved with a single temperature sensor at x=1, the right end of the domain.

2. PROBLEM DESCRIPTION

The problem being studied in this paper is a linear one-dimensional conduction problem. The properties are assumed to be constant with temperature and distance. The governing equation for this problem is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{1}$$

The geometry of the problem is a plate of constant thermal diffusivity as shown in Figure 1. At the left end of the plate a heat flux is applied which varies with time, and the right end is kept insulated.

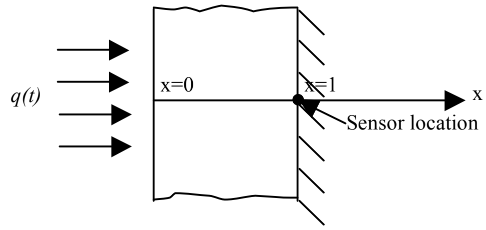

Figure 1. Geometry for the one dimensional direct problem.

Thus the boundary and initial conditions are given by eqns. (2) and (3), respectively.

$$\frac{\partial T}{\partial t}\Big|_{(0,t)} = q(t) \qquad \frac{\partial T}{\partial t}\Big|_{(1,t)} = 0 \tag{2}$$

$$T(x,0) = g(x) \tag{3}$$

The Crank Nicholson finite difference scheme was used to solve the problem numerically. Using this scheme the finite difference form of eqn. (1) can be obtained as

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \frac{1}{2}\alpha\left(\frac{T_{i-1}^{n+1} + T_{i+1}^{n+1} - 2T_i^{n+1}}{(\Delta x)^2} + \frac{T_{i-1}^n + T_{i+1}^n - 2T_i^n}{(\Delta x)^2}\right) \tag{4}$$

Rearranging the above equation so that all the unknowns are on the left hand side of the equation gives

$$-\lambda T_{i-1}^{n+1} + 2(1+\lambda)T_i^{n+1} - \lambda T_{i+1}^{n+1} = \lambda T_{i-1}^n + 2(1-\lambda)T_i^n + \lambda T_{i+1}^n \tag{5}$$

In eqn. (5) λ is $\frac{\alpha \cdot (\Delta t)}{(\Delta x)^2}$ and *i* varies from 0 to N, where N is the number of grid points in the domain $[0,1]$.

Equation (5) is solved along with the discritized boundary conditions for the temperature field at time step *n+1* using the temperature field at time step *n*, and using the tri-diagonal matrix algorithm. Benchmark test cases as given in [1] were run to validate the direct problem solution.

Case 1. A constant heat flux is applied at x=0 and x=1 is insulated. The initial temperature of the plate is taken as $T(x,0) = 0.0$. The temperature response at various locations in the plate is shown in Figure 2 and is found to be in accordance with those shown in [1].

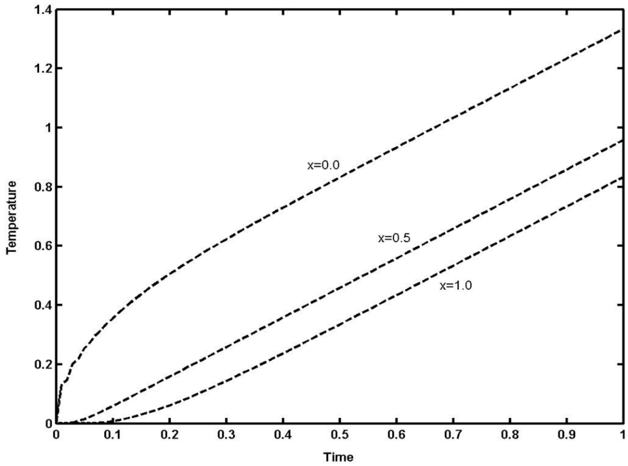

Figure 2. Temperature response at various locations in the plate due to a constant heat flux.

Case 2. A triangular heat flux that varies according to eqn. (6) is applied at x=0, and x=1 is insulated. Initial temperature is $T(x,0) = 0.0$. The temperature response for this case is shown in Figure 3.

$$q(t) = \begin{cases} t & 0 \le t<0.6 \\ 1.2 - t & 0.6 \le t<1.2 \\ 0 & 1.2 \le t<1.8 \end{cases} \qquad (6)$$

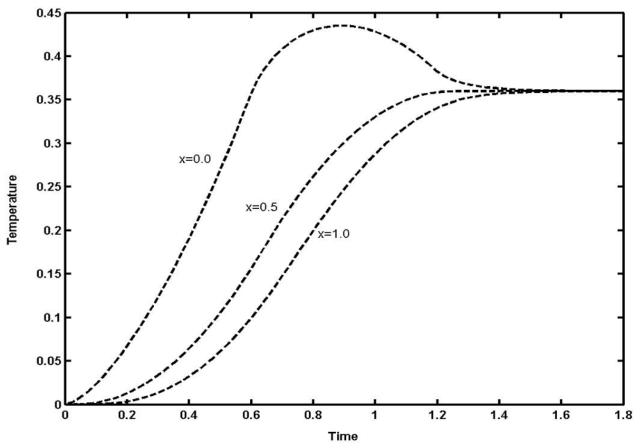

Figure 3. Temperature response at various locations in the plate due to triangular heat flux.

These results from the direct solution of the problem validate the code employed to solve the problem numerically.

3. INVERSE PROBLEM

The inverse problem of estimating the input heat flux when temperature measurements inside the body are known is investigated in this paper. When the sensor location is farthest from the heat source/unknown boundary, it presents a challenge to the inverse algorithm. This is due to damping and lagging effects as explained in [1]. In the inverse problem considered in this paper, the heat flux *q(t)* is unknown, To make up for the lack of this information, temperature measurements from a sensor placed at x=1 are known. Thus T_j^{meas} for all $j = 1$ to N are known at x=1. The boundary condition at x=1 (insulated) and the initial condition remain the same for the inverse problem. Thus the goal of the inverse algorithm is to find the input heat flux, *q(t)*, for which the evaluated temperature at x=1 equals the measured temperature. For this purpose, an optimization based approach using an EA is used in this paper.

4. EVOLUTIONARY ALGORITHMS

Evolutionary algorithms (EAs) are search and optimization methods based on the concept of the survival of the fittest. The main operators in an EA are the selection, crossover and mutation operators. A population of candidate solutions is generated at random, which are stored in an appropriate data structure. An array of double precision numbers is used in this EA. Selection is based on the fitness of the candidate solutions, i.e. those with higher fitness values have a higher probability of selection. The chromosome is an array of double precision numbers of length equal to the number of time steps being considered. The primary advantages of EAs in this application are the ability to identify multiple good solutions which directly supports the engineering decision making process and the ability to avoid trapping these proposed solutions at local optima.

Fitness function

The fitness evaluation function is given by eqn. (7), which is the Tikhonov functional

$$f = \sum_{j=1}^{j=N} (T_j^{cand} - T_j^{meas})^2 + \beta \sum_{j=1}^{j=N} (q_j - q_{j-1})^2 \tag{7}$$

This fitness function has to be minimized, so it is converted into a maximization function by transforming it into the following form, as shown in [5].

$$f' = \frac{1}{f + 0.01} \tag{8}$$

Thus, the minimization problem has been converted into a maximization problem using the transformation in eqn. (8). A population of 32 candidate solutions is used in the EA to solve this inverse problem. The population is initialized randomly. Since the initial population is random, the chromosome initially holds random values that look very different from the required heat flux profile. In transient conduction, the initial heat flux values have to be found correctly before the later flux values are found. For instance, having the correct values for heat flux in the last 10 time steps and incorrect values in the first 10 still gives the chromosome a very low fitness value. This means that the values in the later part of the chromosome do not have their own fitness values but depend on the values of the first part of the chromosome. This is called epistasis. During the evolutionary process, giving more importance to the locations in the chromosome that can be found independently is necessary to solve the problem of epistasis. For this purpose a weight function is multiplied to the first term in eqn. (7). For the inverse problem involving heat conduction, the weight function given by eqn. (9) has been used.

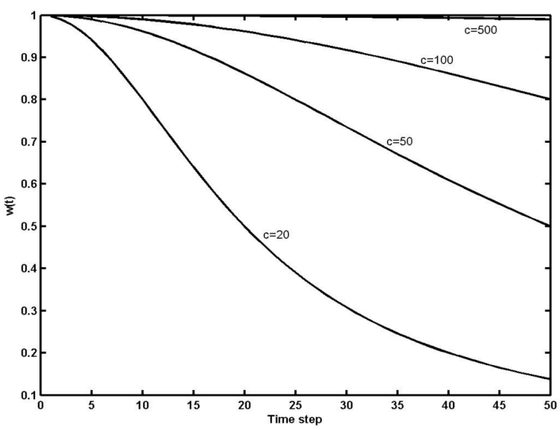

Figure 4. Variation of weights with time step for different values of c.

In eqn. (9), c is a constant that can be set depending on the problem being solved. This weight function is multiplied to the first term in eqn. (7) to obtain a new fitness function. Figure 4 shows how the weights vary

$$w(t) = \frac{1}{(t/c)^2 + 1} \tag{9}$$

as a function of t i.e. time steps. It can be seen that increasing the value of c relaxes the importance of earlier errors as the time increases. Initially this kind of weight function drives the EA to give more attention to the initial heat fluxes because without estimating these correctly, it would be difficult for the EA to estimate the later fluxes correctly because of epistasis.

Crossover operator

Uniform two point crossover has been used to exchange genetic material between the candidate solutions. Two locations are selected at random on the two parents selected for crossover and are swapped at the selected locations.

Mutation operator

The mutation operator is necessary to add new genetic material into the population. The probability of mutation in this EA is increased linearly with the number of mating events. The mutation process constitutes selecting a location on the string at random and modifying the value at that location. This is done by taking a weighted average of the heat flux values of the past and future three times at the selected location. So, if the selected location is *l*, then the new value at *l*, T_l , is given by

$$T_l = \{-2T_{l-3} + 3T_{l-2} + 6T_{l-1} + 7T_l + 6T_{l+1} + 3T_{l+2} - 2T_{l+3}\}/21 \tag{10}$$

At the locations where *l-3, l-2, l-1, l+1, l+2* and *l+3* are not defined, the values are taken to be zero. This kind of mutation operator helps in making the discrete values in the string more continuous. To help the EA escape from local optima, after every 500 mating events a randomly selected value in the chromosome is mutated with a random value.

5. RESULTS

Two test cases were run to check the performance of the EA in estimating correctly the input heat flux *q(t)*. The direct problem was solved for the two cases, and the temperature response at x=1 from the solutions was used as experimental data for the inverse problem.

Linearly varying heat flux

Data for the inverse problem was collected by solving the governing equations by applying a linearly varying heat flux at x=0, given by

$$q(t) = t,\ \ 0 \le t \le 0.2 \tag{11}$$

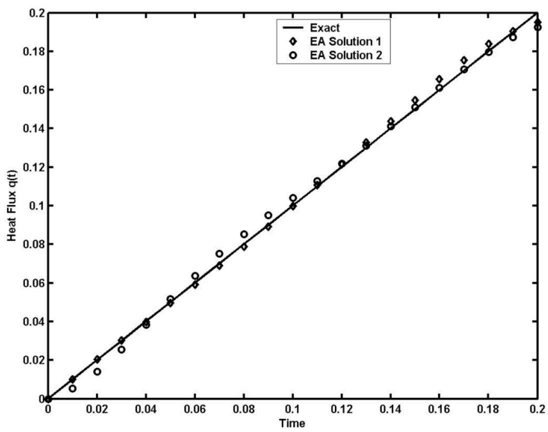

Figure 5. Solutions obtained form the EA for linearly varying flux.

Triangular variation of heat flux

A triangular heat flux variation, given by eqn. (12), is used to generate data for the inverse problem. This is one of the most stringent test cases in an IHCP. The goal of the EA is to match as closely as possible this input profile, using the data from the direct problem solution.

$$q(t) = \begin{cases} t & 0 \le t < 0.1 \\ 0.1 - t & 0.1 \le t < 0.2 \\ 0 & t \ge 0.2 \end{cases} \tag{12}$$

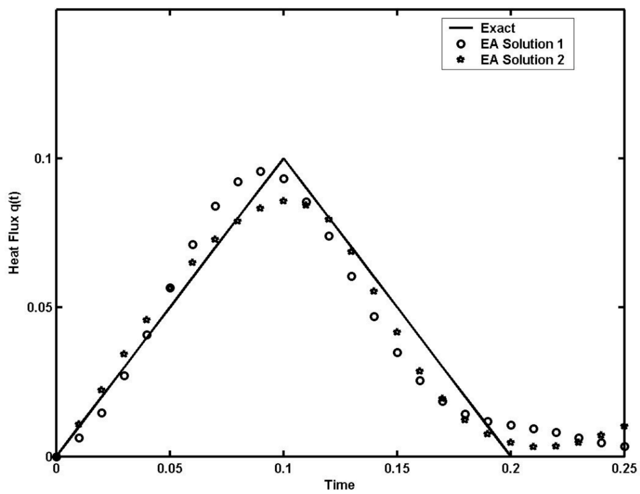

Figure 6. Solutions obtained form the EA for the triangular variation of flux.

It can be seen from Figures (5) and (6) that the EA matches the input heat flux well. These solutions have been obtained for the case where the weights *w*, have been used in conjunction with eqn. (7). Figures (7) and (8) show the best solutions obtained at various stages of the evolutionary process.

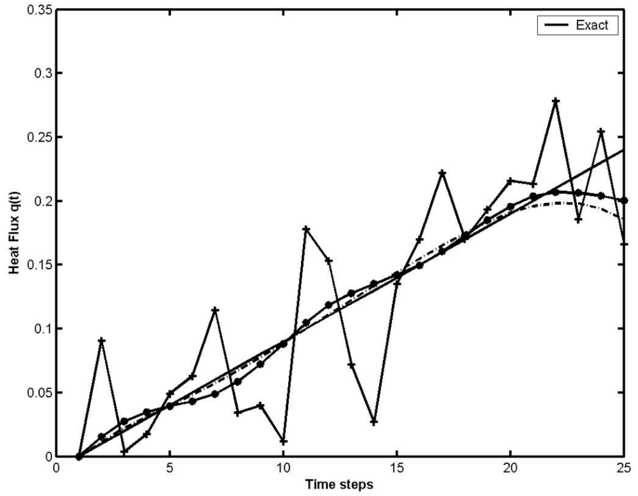

Figure 7. Best candidate solutions at various stages in the EA, linearly varying heat flux after (+) 1, (*) 15 and (-.-) 40 thousand mating events respectively.

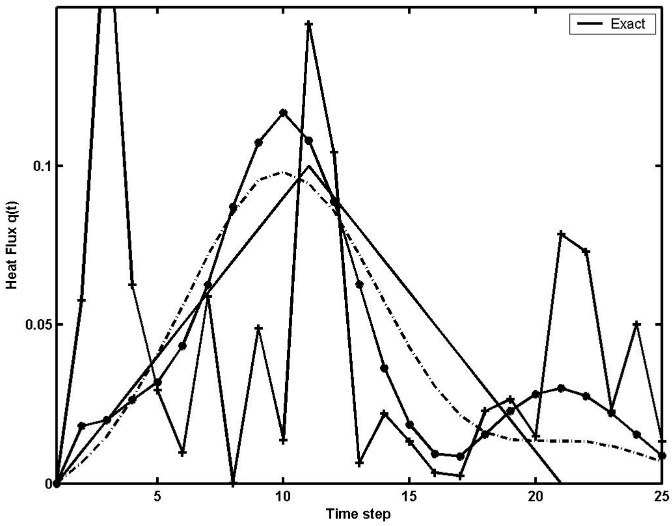

Figure 8. Best candidate solutions at various stages in the EA, using triangular variation of heat flux after (+) 1, (*) 15 and (-.-) 40 thousand mating events respectively.

It was found that using the exact number of time steps resulted in the solution being inaccurate at later time steps, and hence more time steps than needed were used in the solution procedure. This occurs because of the lag in temperature response at the sensor at the farthest end of the domain. Physically, it takes more time for the information at the left end to be transmitted to the right end of the plate. Hence, for estimating heat flux at the right end of the plate, more time steps than necessary have to be considered for correctly estimating the heat flux. This resulted in accurate solutions for the required number of time steps.

Comparison of fitness

The scaled fitness values for the two situations, i.e. (a) weights not used in fitness evaluation and (b) weights used in fitness evaluation, are compared in Figures (9) and (10). These fitness values were recorded from the 1000th mating event onwards. It can be seen that using weights in the fitness evaluations has a distinct advantage to find better fitness candidate solutions in the initial stages of the evolutionary process. The number of mating events to solution is fewer in the case when weights are used.

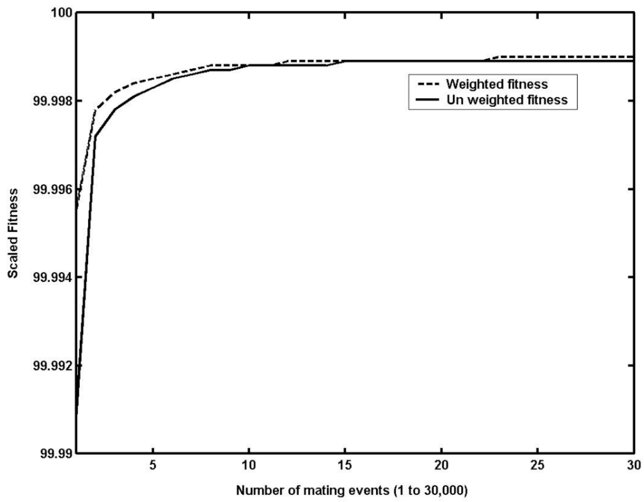

Figure 9. Comparison of weighted and unweighted fitness values for the linear heat flux case.

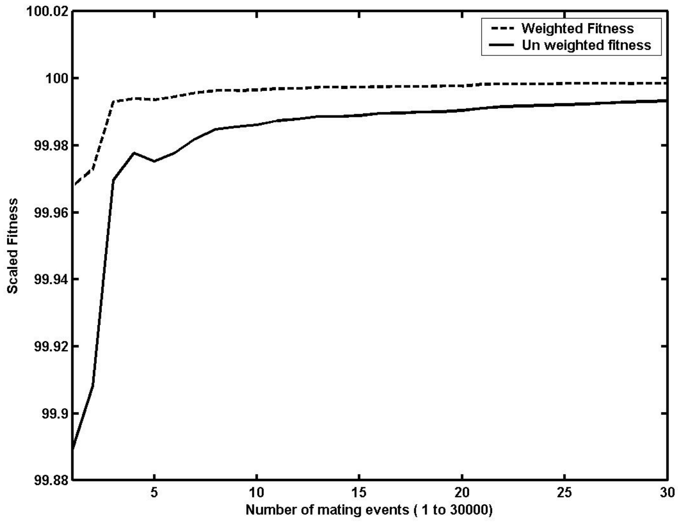

Figure 10. Comparison of weighted and unweighted fitness values for the triangular heat flux case.

Values of c used for the linear and triangular heat flux cases are 10 and 100 respectively. It is evident from Figures 9 and 10 that the increase in fitness is much more in the triangular heat flux case. Thus, multiplying the first term of the Tikhonov functional by a weight function allows the EA to search the solution space for a solution at the initial time steps in a better manner before moving onto the later time steps. Figure 11 shows the variation of average fitness of the population and maximum fitness averaged over 30 runs of the EA. This figure shows that the population is evolving to the maximum fitness value. This data is taken for the linear heat flux case and plotted for the first 1500 mating events.

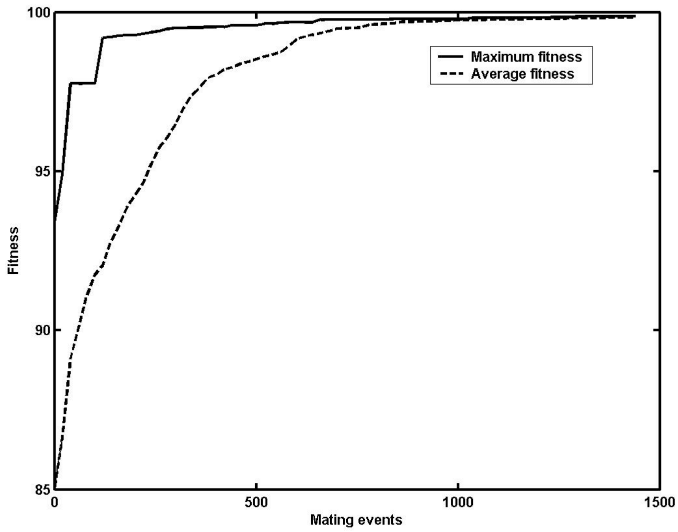

Figure 11. Comparison of average and maximum fitness values.

6. CONCLUSIONS AND FUTURE WORK

In this paper, a one-dimensional IHCP was solved by minimizing a modified form of Tikhonov's functional using an EA. The solutions obtained from the EA are accurate. It was found that this method resulted in better performance of the EA by looking at the variation of fitness with the number of mating events. These preliminary results show promise for use in more complicated problems. The EA will be extended for non-linear heat conduction problems as well in which the thermal properties vary with temperature. Performance of the algorithm in the presence of noise in the measured temperature data also has to be evaluated.

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